Computer Graphics

Lecture 13

Two dimensional Geometric transformation

With the procedures for displaying output primitives and their attributes, we can create variety of pictures and graphs. In many applications, there is also a need for altering or manipulating displays. Design applications and facility layouts are created by arranging the orientations and sizes of the component parts of the scene. And animations are produced by moving the "camera" or the objects in a scene along animation paths. Changes in orientation, size, and shape are accomplished with geometric transformations that alter the coordinate descriptions of objects. The basic geometric transformations are translation, rotation, and scaling. Other transformations that are often applied to objects include reflection and shear. We first discuss methods for performing geometric transformations and then consider how transformation functions can be incorporated into graphics packages.

Basic Transformations

Here, we first discuss general procedures for applying translation, rotation, and scaling parameters to reposition and resize two-dimensional objects

Translation

A translation is applied to an object by repositioning it along a straight-line path from one coordinate location to another. We translate a two-dimensional point by adding translation distances, t_x and t_y, to the original coordinate position (x, y) to move the point to a new position (x', y') (Fig. 5-1).

$$x' = x + t_x, \qquad y' = y + t_y \tag{5.1}$$

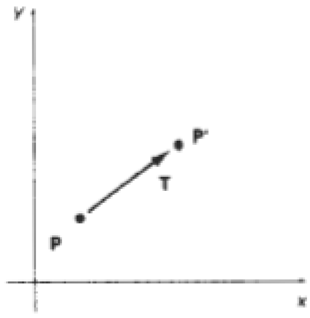

Figure 5-1
Translating a point from position **P** to position **P'** with translation vector **T**.

The translation distance pair (t_x, t_y) is called a translation vector or shift vector. We can express the translation equations 5.1 as a single matrix equation by using column vectors to represent coordinate positions and the translation vector.

$$\mathbf{P} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \quad \mathbf{P'} = \begin{bmatrix} x'_1 \\ x'_2 \end{bmatrix}, \quad \mathbf{T} = \begin{bmatrix} t_x \\ t_y \end{bmatrix} \tag{5-2}$$

This allows us to write the two-dimensional translation equations in the matrix form:

$$P' = P + T \tag{5.3}$$

Sometimes matrix-transformation equations are expressed in terms of coordinate row vectors instead of column vectors. In this case, we would write the matrix representations as P = [x y] and T = [t_x, t_y]. Since the column-vector representation for a point is standard mathematical notation, and since many graphics packages, for example, GKS and PHIGS, also use the column-vector representation, we will follow this convention

Translation is a rigid-body transformation that moves objects without deformation. That is, every point on the object is translated by the same amount. A straight Line segment is translated by applying the transformation equation 5-3 to each of the line endpoints and redrawing the line between the new endpoint positions. Polygons are translated by adding the translation vector to the coordinate position of each vertex and regenerating the polygon using the new set of vertex coordinates and the current attribute settings. Figure 5-2 illustrates the application of a specified translation vector to move an object from one position to another.

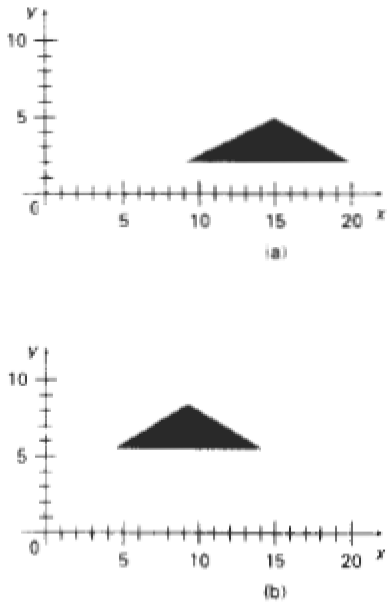

Figure 5-2
Moving a polygon from position (a) to position (b) with the translation vector (-5.50, 3.75).

Similar methods are used to translate curved objects. To change the position of a circle or ellipse, we translate the centre coordinates and redraw the figure in the new location. We translate other curves (for example, splines) by displacing the coordinate positions defining the objects, then we reconstruct the curve paths using the translated coordinate points.

Rotation

A two-dimensional rotation is applied to an object by repositioning it along a circular path in the xy plane. To generate a rotation, we specify a rotation angle θ and the position (x_r, y_r) of the rotation point (or pivot point) about which the object is to be rotated (Fig. 5-3). Positive values for the rotation angle define counter clockwise rotations about the pivot point, as in Fig. 5-3, and negative values rotate objects in the clockwise direction. This transformation can also be described as a rotation about a rotation axis that is perpendicular to the xy plane and passes through the pivot point.

We first determine the transformation equations for rotation of a point position P when the pivot point is at the coordinate origin. The angular and coordinate relationships of the original and transformed point positions are shown in Fig. 5-4. In this figure, r is the constant distance of the point from the origin, angle ∅ is the original angular position of the point from the horizontal, and θ is the rotation angle. Using standard trigonometric identities, we can express the transformed coordinates in terms of angles θ and ∅ as

$$\begin{aligned} x' &= r\cos(\phi + \theta) = r\cos\phi\cos\theta - r\sin\phi\sin\theta \\ y' &= r\sin(\phi + \theta) = r\cos\phi\sin\theta + r\sin\phi\cos\theta \end{aligned} \tag{5-4}$$

The original coordinates of the point in polar coordinates are

$$x = r\cos\phi, \qquad y = r\sin\phi \tag{5-5}$$

Substituting expressions 5-5 into 5-4, we obtain the transformation equations for rotating a point at position (x, y) through an angle θ about the origin:

$$\begin{aligned} x' &= x\cos\theta - y\sin\theta \\ y' &= x\sin\theta + y\cos\theta \end{aligned} \tag{5-6}$$

With the column-vector representations 5.2 for coordinate positions, we can write the rotation equations in the matrix form:

$$P' = R \cdot P \tag{5.7}$$

Where the rotation matrix is

$$\mathbf{R} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \tag{5-8}$$

When coordinate positions are represented as row vectors instead of column vectors, the matrix product in rotation equation 5-7 is transposed so that the transformed row coordinate vector [x' y'] is calculated as

$$\begin{aligned} \mathbf{P}'^{T} &= (\mathbf{R} \cdot \mathbf{P})^{T} \\ &= \mathbf{P}^{T} \cdot \mathbf{R}^{T} \end{aligned}$$

where P^T = [x y], and the transpose R^T of matrix R is obtained by interchanging rows and columns. For a rotation matrix, the transpose is obtained by simply changing the sign of the sine terms.

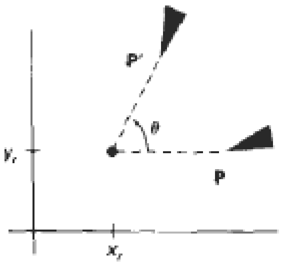

Figure 5-3
Rotation of an object through angle θ about the pivot point (x_r, y_r).

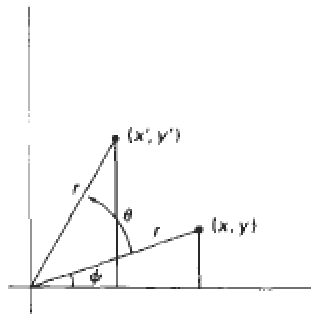

Figure 5-4
Rotation of a point from position (x, y) to position (x', y') through an angle θ relative to the coordinate origin. The original angular displacement of the point from the x axis is ϕ.

Rotation of a point about an arbitrary pivot position is illustrated in Fig. 5-5. Using the trigonometric relationships in this figure, we can generalize Eqs. 5-6 to obtain the transformation equations for rotation of a point about any specified rotation position (x_r, y_r):

$$
\begin{aligned}
x' &= x_r + (x - x_r)\cos\theta - (y - y_r)\sin\theta \\
y' &= y_r + (x - x_r)\sin\theta + (y - y_r)\cos\theta
\end{aligned}
\tag{5-9}
$$

These general rotation equations differ from Eqs. 5-6 by the inclusion of additive terms, as well as the multiplicative factors on the coordinate values. Thus, the matrix expression 5-7 could be modified to include pivot coordinates by matrix addition of a column vector whose elements contain the additive (translational) terms In Eqs. 5-9.

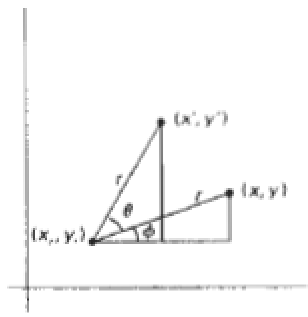

Figure 5-5
Rotating a point from position (x, y) to position (x', y') through an angle θ about rotation point (x_r, y_r).

As with translations, rotations are rigid-body transformations that move objects without deformation. Every point on an object is rotated through the same angle. A straight line segment is rotated by applying the rotation equations 5-9 to each of the line endpoints and redrawing the line between the new endpoint positions. Polygons are rotated by displacing each vertex through the specified rotation angle and regenerating the polygon using the new vertices. Curved lines are rotated by repositioning the defining points and redrawing the curves. A circle or an ellipse, for instance, can be rotated about a non-central axis by moving the centre position through the arc that subtends the specified rotation angle. An ellipse can be rotated about its centre coordinates by rotating the major and minor axes.